Simple harmonic and circular motions

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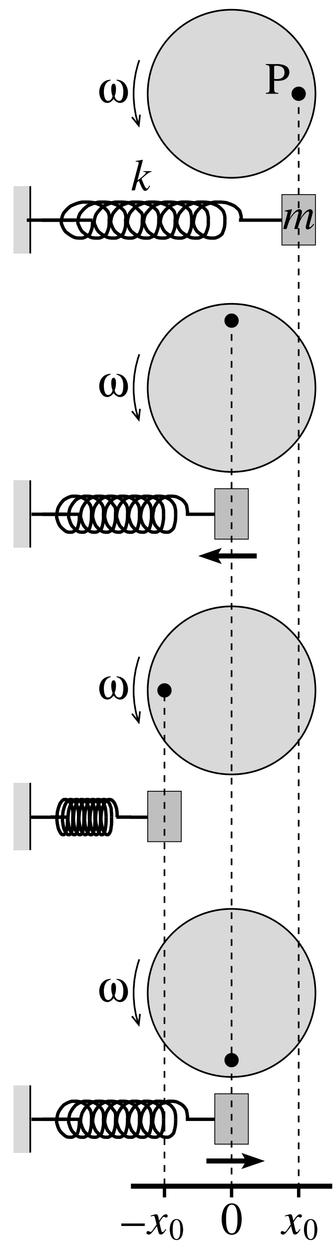

A mass m oscillating on a Hooke's law spring ($F = -kx$) with spring constant k undergoes *simple harmonic motion* governed by:

$$x(t) = A\cos\omega_0 t + B\sin\omega_0 t, \qquad (1)$$

where $\omega_0^2 = k/m$; A, B set by *initial conditions*.

Thus, if at $t = 0$, m is pulled to $x = x_0$ and released from rest (top panel),

$$x(0) = A = x_0; \qquad \dot{x}(0) = B\omega_0 = 0,$$

and equation (1) becomes:

$$x(t) = x_0 \cos\omega_0 t. \qquad (2)$$

Consider now a point P undergoing *circular motion* (radius $r = x_0$) at constant angular speed ω (and thus, $\theta = \omega t$). If at $t = 0$, P is at $(x_0, 0)$ (top panel), then the x-component of its position is:

$$x(t) = r\cos\theta = x_0\cos\omega t,$$

exactly the same as equation (2) when $\omega = \omega_0$.

Thus, as viewed "edge-on" from the $-y$-axis, circular motion is *identical* to simple harmonic motion.

This YouTube video is a nice illustration of this equivalence.