Assignment 2, PHYS 2302

assigned Thursday, September 29; due Thursday, October 6

Problem 1 For each of the following first order ODEs, find $y(x)$ using separation of variables:

a) $(x+1)\dfrac{dy}{dx} - y \;=\; 1;$

b) $(x^2-1)\cot y\dfrac{dy}{dx} \;=\; 1;$

c) $x\dfrac{dy}{dx} - y - x^2(1-x) \;=\; 0.$

Hint: Turns out that as written, the ODE in part c is not separable. However, if you rearrange the equation as:

$$\frac{xy' - y}{x^2} \;=\; 1 - x,$$

you might recognise $(xy'-y)/x^2 \;=\; (y/x)'$ as a *perfect derivative*. Hmmmm, this might suggest a substitution of, say, $z = y/x\ldots$

Problem 2 (FC 2.1) Find $v(t)$ and $x(t)$ for a mass, m, starting from rest at $x = 0$ and $t = 0$, subject to the forces:

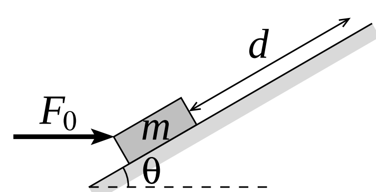
a) $F_x = F_0 + ct;$

b) $F_0\sin(ct)$; and

c) F_0e^{ct}.

Problem 3 Starting from rest, a block of mass m is pushed up a ramp with inclination angle θ by a force F_0 applied horizontally. After m moves up the ramp a distance d, its speed is v upward along the ramp.

Using the work-kinetic theorem and the following data: $m = 1.00\,\mathrm{kg}$; $g = 9.81\,\mathrm{m\,s^{-2}}$; $F_0 = 12.0\,\mathrm{N}$; $\theta = 30°$; $d = 0.270\,\mathrm{m}$; and $v = 1.26\,\mathrm{m\,s^{-1}}$, find a numerical value (to three significant figures) for the coefficient of kinetic friction, μ_k, between the block and the ramp surface.



Hint: As derived in class so far, the W-K theorem is for rectilinear (1-D) motion. Thus, if the displacement is along the x axis and the x-component of a constant force $\vec{F}$ is F_x, the work done by $\vec{F}$ along $0 \le x \le d$ is given by:

$$W_F = \int_0^d F_x \, dx = F_x \int_0^d dx = F_x d. \tag{1}$$

Problem 4 (FC 2.8, 2.15)

a) A mass, m, moves with velocity $v(x) = b/x^3$, where $b > 0$ is a constant. Find $F(x)$ acting on the mass.

b) Taking into account both the linear and quadratic terms in the air drag force,

$$D = -c_1 v - c_2 v^2, \tag{1}$$

(*e.g.*, equation 2.4.3, p. 69, ed. 7 of F&C), show that the terminal speed of a falling object of mass m is given by:

$$v_{\mathrm{t}} = \sqrt{\frac{mg}{c_2} + \left(\frac{c_1}{2c_2}\right)^2} - \frac{c_1}{2c_2}.$$

Problem 5 (FC 2.18, modified) The force acting on a particle of mass m is given by $F = \alpha\sqrt{v}\,t$, where $\alpha > 0$ is a constant. If m passes through the origin ($x = 0$) with speed v_0 at $t = 0$, find $x(t)$, the position of m as a function of time.