Assignment 6, PHYS 2302

assigned Thursday, November 3; due Thursday, November 17 (two weeks)

Problem 1 (FC 4.2) By taking their curls, determine which of the following forces are conservative.

a) $\vec{F} = (x, y, z)$

b) $\vec{F} = (y, -x, z^3)$

c) $\vec{F} = (y, x, z^3)$

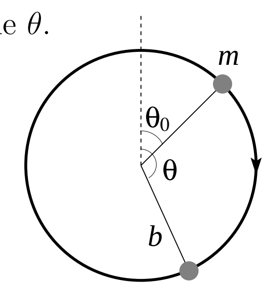
d) $\vec{F} = -kr^{-n}\hat{e}_r$, where $\hat{e}_r$ is the unit radial vector in spherical polar coordinates.

Problem 2 (FC 4.5 modified) Let C_1 be the path on the x-y plane that joins $(0, 0)$ and $(1, 1)$ directly by the parabolic path $y = \sqrt{x}$, and let C_2 be the path that joins $(0, 0)$ to $(0, 1)$ by the line $x = 0$, and then $(0, 1)$ to $(1, 1)$ by the line $y = 1$.

a) Evaluate $\int \vec{F} \cdot d\vec{r}$ for $\vec{F} = xy\hat{\imath} + \frac{1}{2}x^2\hat{\jmath}$ for each of the paths C_1 and C_2, and verify that $\vec{F}$ is conservative.

b) Evaluate $\int \vec{F} \cdot d\vec{r}$ for $\vec{F} = xy\hat{\imath} + x^2\hat{\jmath}$ for each of the paths C_1 and C_2, and show that $\vec{F}$ is not conservative.

Problem 3 (FC 4.22) In the diagram, a bead of mass m slides on a smooth circular wire of radius b starting from rest at position angle, θ_0, to position angle θ.



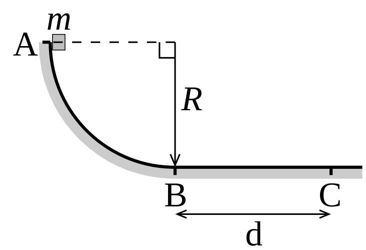
a) Find the speed of m at θ.

b) Find the normal force exerted by the wire on m at θ.

c) Find the values of θ_0 and θ where N a maximum, and find N_{max}.

Problem 4 The figure shows a track consisting of a quarter circle of radius R attached seamlessly at point B to a straight portion of indefinite length. A mass, m, is released from rest at point A, slides down the track to B, and comes to rest at C, a distance d from B.

If the coefficient of kinetic friction between m and the entire track is μ_k, use the W-K theorem to find the *average* frictional force along the curved portion of the track, AB. Recall that by definition, the average value of a quantity, q, over a path P of length L is,



$$\langle q \rangle_{\mathrm{P}} \equiv \frac{1}{L} \int_{\mathrm{P}} q(s)\, ds, \tag{1}$$

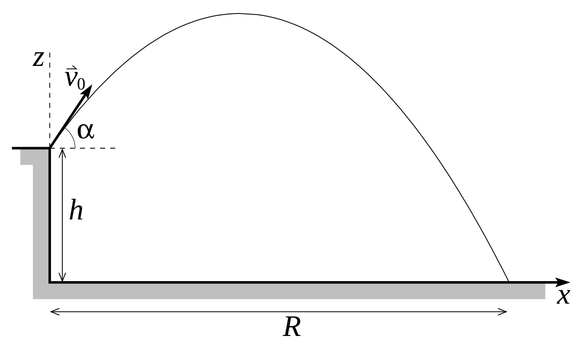
where ds is an infinitesimal displacement along the path.

Problem 5 (FC 4.9) A cannon is situated at the edge of a bluff a height h over a plain below, and fires a cannon ball with a muzzle speed v_0.

a) Ignoring air resistance, show that the elevation angle, α, required to achieve maximum range across the plain is given by:

$$\sin^2\alpha = \frac{1}{2} \frac{v_0^2}{v_0^2 + gh}.$$

b) What is the maximum range, R_{max}, of the cannon in terms of v_0, h, and g?

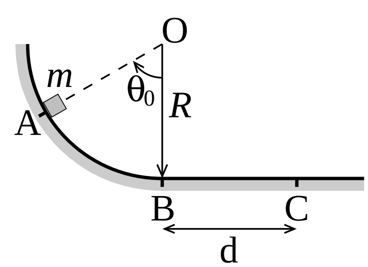


Problem 6 (Challenge problem, 10 point bonus) A track with coefficient of kinetic friction μ_k *over its entire length* consists of a quarter circle of radius R attached seamlessly at point B to a straight portion of arbitrary length. A mass, m, is placed on the curved portion at point A such that angle AOB = θ_0, and released from rest. It slides down the track past point B, and comes to rest at point C, a distance d beyond B.

a) From Newton's 2nd Law, show that the centripetal acceleration, a_{cp}, of m as it slides along the quarter circle is given by the 1st order ODE:

$$\frac{da_{\mathrm{cp}}}{d\theta} - 2\mu_k a_{\mathrm{cp}} = 2g(\mu_k \cos\theta - \sin\theta), \tag{1}$$

where $\theta_0 > \theta > 0$ is the angle mOB.



Hint: You may find it useful to know that the tangential acceleration and centripetal acceleration can be related as follows:

$$a_{\mathrm{tan}} = \frac{dv_{\mathrm{tan}}}{dt} = \frac{d\theta}{dt}\frac{dv_{\mathrm{tan}}}{d\theta} = -\frac{v_{\mathrm{tan}}}{R}\frac{dv_{\mathrm{tan}}}{d\theta} = -\frac{1}{2}\frac{d}{d\theta}\left(\frac{v_{\mathrm{tan}}^2}{R}\right) = -\frac{1}{2}\frac{da_{\mathrm{cp}}}{d\theta}.$$

Here, I've used $\dot{\theta} = -\dfrac{v_{\mathrm{tan}}}{R}$ since v_{tan} increases in the direction of decreasing θ.

b) Solve equation (1) for $a_{\mathrm{cp}}(\theta)$ by finding the homogeneous solution, adding on a particular solution, and then applying the boundary condition (at $\theta = \theta_0$, $a_{\mathrm{cp}} = 0$). After some algebra, show that at B ($\theta = 0$):

$$v_{\mathrm{B}}^2 = a_{\mathrm{cp,B}} R = \frac{2gR}{1+4\mu_{\mathrm{k}}^2}\left[\left(1-2\mu_{\mathrm{k}}^2\right)\left(1-\cos\theta_0 e^{-2\mu_{\mathrm{k}}\theta_0}\right) - 3\mu_{\mathrm{k}}\sin\theta_0 e^{-2\mu_{\mathrm{k}}\theta_0}\right] \qquad (2)$$

c) Use the W-K theorem between B and C to find the distance, d, where m comes to rest.