Assignment 7, PHYS 2302

assigned Thursday, November 17; due Thursday, December 1 (two weeks)

Problem 1 (FC 4.12) A baseball player, who can throw a ball horizontally more easily than vertically, throws a ball at speed $v_0 \cos(\alpha/2)$ where $v_0 = 25\ \mathrm{m\,s^{-1}}$ is his speed when thrown horizontally, and $0 < \alpha < \pi/2$ is the elevation angle.

a) Find the value of α that maximises the height, H, and evaluate H_{max}.

b) Find the value of α that maximises the range, R, and evaluate R_{max}.

Problem 2 (FC 4.15) In class, we considered a projectile of mass m and velocity $\vec{v}$ experiencing a drag force given by $\vec{D} = -m\gamma\vec{v}$, where γ is the coefficient of linear air drag. By solving the differential equations stemming from Newton's 2nd Law, we found the projectile *path* to be:

$$\vec{r}(t) = \frac{1}{\gamma}\left[\left(\vec{v}_0 + \frac{g}{\gamma}\hat{k}\right)\left(1 - e^{-\gamma t}\right) - gt\hat{k}\right], \tag{1}$$

where $\vec{v}_0$ is the projectile's initial velocity and $\hat{k}$ is a unit vector in the vertical (z) direction.

a) Show that the *trajectory* of the projectile is given by:

$$z(x) = \frac{x}{v_{0x}}\left(v_{0z} + \frac{g}{\gamma}\right) + \frac{g}{\gamma^2}\ln\left(1 - \frac{\gamma x}{v_{0x}}\right), \tag{2}$$

where x is the horizontal direction along which the projectile travels, and $\vec{v}_0 = (v_{0x}, v_{0z})$.

b) Show that the range of the projectile, R (value of $x > 0$ when $z = 0$), is given by:

$$R = R_0\left(1 - \frac{4\gamma v_0}{3g}\sin\alpha + \mathcal{O}(\gamma^2)\right),$$

where $R_0 = (v_0^2 \sin 2\alpha)/g$ is the range of the particle for $\gamma = 0$ (no air drag), α is the inclination angle of the projectile at the beginning of its trajectory, and $\mathcal{O}(\gamma^2)$ represents ignored terms of power γ^2 or higher.

Problem 3 (FC 4.17 modified) A mass m is suspended by six massless springs aligned along the Cartesian axes (*e.g.*, Fig. 4.4.1 of eds. 6 and 7) with spring constants $(k_x, k_y, k_z) = k\left(\frac{9}{4}, 1, 4\right)$, where $k = \pi^2 m/2$ (and thus, with two springs in the y-direction, for example, $F_y = -2k_y y = -\pi^2 m y$).

a) If, at $t = 0$, $\vec{r} = (1, -1, 1)$ and $\vec{v} = \vec{0}$, find $\vec{r}(t)$.

b) Does m ever retrace its path and, if so, find the smallest value of t where m returns to its initial conditions.

Problem 4 (FC 4.18) For a 2-D isotropic oscillator, we showed in class that the path taken by a mass, m, is an ellipse, given by:

$$\frac{x^2}{A^2} - xy\frac{2\cos\Delta}{AB} + \frac{y^2}{B^2} = \sin^2\Delta, \tag{1}$$

(*e.g.*, equation 4.4.10 in ed. 7 of the text), where A, B, and Δ were defined in class.

Show that the major axis of this ellipse is inclined relative to the x-axis by an angle ψ, where:

$$\tan 2\psi = 2\frac{AB\cos\Delta}{A^2 - B^2}.$$

Problem 5 (FC 4.20) An electron moves in the electromagnetic field $\vec{E} = E\hat{\jmath}$, $\vec{B} = B\hat{k}$, where E and B are constant. If $\vec{r}_0 = 0$ and $\vec{v}_0 = v_0\hat{\imath}$, show that the resulting particle path is a cycloid parameterised by t and given by:

$$x(t) = a\sin\omega t + bt; \qquad y(t) = a(1-\cos\omega t); \qquad z(t) = 0,$$

where $\omega = eB/m$, $b = E/B$, and $a = (v_0 - b)/\omega$.