PHYS 2302 FIRST PRACTISE MIDTERM

Instructions

There are three problems on this exam, each worth 20 points. Do as much of this exam as you can; your mark will be the sum of all your points taken out of 45.

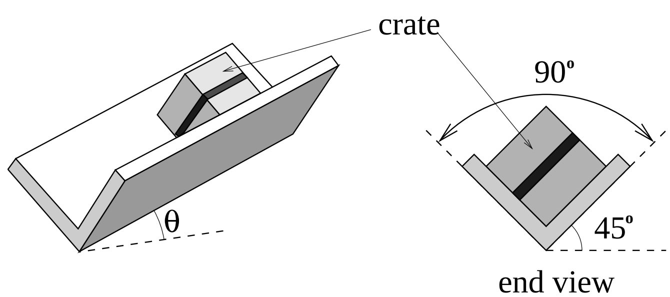
This is a closed-book exam; notebooks, textbooks, *etc.*, are not permitted and all electronic devices should be put away.

Formulæ are provided on the last page of the exam. Including the formula sheet, there are three pages to this exam (fourth page blank). You may tear off the formula sheet from the exam for ready reference if you like.

Solutions are to be written in the exam booklet provided. This exam, including the formula sheet, must be turned in with your solutions.

You have 75 minutes.

Problem 1. Crate sliding down a V-ramp: A crate slides down a right-angle trough inclined to the horizontal at an angle θ as shown in the figure. If the coefficient of kinetic friction between the sides of the crate and the surface of the trough is μ_{k}, find the acceleration of the crate.



Hint: Normal forces are exerted on the crate by both sides of the trough.

Problem 2. Sliding block: A metal block slides along a horizontal surface lubricated with a heavy oil that imposes on the block a resistive force given by:

$$F(v) = -cv^{3/2}, \tag{1}$$

where v is the speed of the block, c is a constant, and the negative sign indicates this force causes the block to slow down.

a) (12 points) If at $x = 0$ the speed of the block is v_0, show that the speed of the block after it slides a distance x is given by,

$$v(x) = \left(\sqrt{v_0} - \frac{cx}{2m}\right)^2 .$$

b) (2 points) Find X, the maximum distance travelled by the block.

c) (6 points) By solving the differential equation,

$$F(v) = m\frac{dv}{dt},$$

determine whether the block reaches X asymptotically (*i.e.*, as $t \to \infty$) or after a finite time, T.

Problem 3. SHO plug and chug: Consider the position of a simple harmonic oscillator as a function of time given by,

$$x(t) = C\cos(\omega_0 t - \phi_0), \qquad (2)$$

where C is the amplitude of oscillation, ω_0 is the angular frequency of oscillation, and ϕ_0 is the phase of oscillation determining where the oscillator is at $t = 0$.

a) (5 points) Show by direct substitution that equation (2) solves the differential equation of motion for a simple harmonic oscillator, as given on the formula sheets.

b) (15 points) If, at $t = 0$, $x(0) = x_0$ and $v(0) = \dot{x}(0) = v_0$, find C and ϕ_0 in terms of quantities given.

FORMULA SHEET

1. Variations of **Newton's Second Law** for rectilinear (1-D) motion:

$$\sum F \,=\, ma \,=\, m\ddot{x} \,=\, m\dot{v} \,=\, m\frac{dv}{dx}v \,=\, \frac{m}{2}\frac{dv^2}{dx}.$$

2. **Frictional forces** (Coulomb model):

$$f_{\mathrm{k}} \,=\, \mu_{\mathrm{k}} N; \qquad f_{\mathrm{s}} \,\leq\, \mu_{\mathrm{s}} N; \qquad D \,=\, c_1 v + c_2 v^2,$$

where f_{k}, f_{s}, and D are the kinetic friction force, static friction force, and "air drag" respectively, N is the normal force, v is the velocity of a particle through the air, and where the remaining quantities are coefficients.

3. **Simple harmonic oscillator (SHO):** Any system of mass m whose differential equation of motion has the form,

$$\frac{d^2x}{dt^2} + \omega_0^2 x \,=\, 0,$$

or whose total energy has the form,

$$E \,=\, K + U \,=\, \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\omega_0^2 x^2 \,=\, \text{constant},$$

describes an undamped, undriven SHO oscillating at angular frequency ω_0 ($\mathrm{rad\,s^{-1}}$).

The solution to either of these equations can be written in three forms:

$$x(t) \,=\, A\cos\omega_0 t + B\sin\omega_0 t \,=\, x_0\cos(\omega_0 t - \phi_0) \,=\, ae^{i\omega_0 t} + be^{-i\omega_0 t},$$

where: $A, B \in \mathbb{R}$ are constants of integration set by initial conditions;
x_0 = amplitude of oscillation;
ϕ_0 = phase of oscillation;
$a, b \in \mathbb{C}$ are constants of integration set by initial conditions.

4. **Solving a first-order ODE by separation of variables**: A first order ordinary differential equation of the form,

$$\frac{dy}{dx} \,=\, \frac{f(x)}{g(y)},$$

where $f(x)$ and $g(y)$ are arbitrary functions is *separable*, and can be rewritten as,

$$g(y)\,dy \,=\, f(x)\,dx \quad \Rightarrow \quad \int g(y)\,dy \,=\, \int f(x)\,dx.$$